



## Technical Note

## Heat transfer through woven textiles

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## ABSTRACT

A mathematical model based on the principles of heat transfer to predict the thermal resistance of fabrics has been presented in this paper. The woven fabric is considered as a system of porous yarns, interlacings between warp and weft yarns and air pores and all the basic weaves can be depicted by this system. The conduction and radiation heat transfer together, was calculated based on the construction parameters of the fabric. The thermal insulation, which is equivalent to the thermal resistance, was predicted with the help of these parameters. The total heat transfer by conduction through each part was calculated using Fourier's equation. Radiation heat transfer through the air pore was calculated with the help of net radiation method. Linear anisotropic scattering was used to model the radiation heat transfer through fibrous media. The total thermal resistance obtained was validated with actual values obtained from a standard thermal resistance measuring instrument.

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## 1. Introduction

Thermal properties of textile materials, especially thermal resistance have always been the major concern when the comfort properties of clothing are characterized. Prediction with the help of statistical methods is based on studying the effects of different fabric and environmental variables on the thermal resistance values, the variables can be independent or in combination with each other. It is difficult to obtain a clear relationship between the individual fabric parameters on the thermal resistance as most of them are interrelated to each other and therefore impossible to separate. The authors have shown that a better relationship can be obtained using response surface methodology when all the constructional parameters are taken together and their collective effect on thermal resistance is observed [1]. Stochastic models like artificial neural networks can be used to predict the thermal resistance to a good degree of precision but it is only possible when all the constructional parameters are considered collectively and a large amount of data is available for training the model [2]. Mechanistic models are based on various mathematical treatments of the heat transfer phenomenon. Although the mechanistic models use a number of simplifications, which sometimes results in individual prediction error, they are equally important as they give an insight into the physics of the heat transfer. Studies on heat transfer through various materials has

been going on since a long time, these include heat transfer through engineering materials and structures like stack of rods and fins, porous materials viz. granular, cellular as well as fibrous insulators. Various researchers have tried to understand the phenomena of heat and mass transfer through fibrous media. Farnworth considered the combined conductive and radiative heat flow through fibrous insulating materials without any convective heat transfer, even in very low density battings [3]. Bankvall [4] considered the heat loss due to conduction and radiation through fibrous insulations in absence of convective heat transfer considering a unit volume of the system consisting of both series and parallel systems of fibres and fluid viz. gas. Ismail et al. [5] gave a theoretical model to predict the thermal conductivity of a plain woven textile fabric. Chang [6] considered a simple model based on the combined series and parallel conduction for the effective conductivity of fluid saturated screens. Daryabeigi [7] worked on the radiation/conduction heat transfer in high porosity, high temperature alumina fibrous insulations.

The treatment of heat transfer through fabric structures is different from fibrous media because the fabric, woven or knitted, comprises of a repeat unit which consists of porous yarns and air spaces. To find the heat transfer through a fabric structure, one has to consider both the systems. With the help of the studies conducted in the past and observations for various porous regions, a mathematical model was formulated based on the principles of heat transfer by conduction and radiation. The model was first applied on plain woven fabrics using simple Fourier's law of conduction and Stefan-Boltzmann's law of radiation [8]. Subsequently, the model was modified to include the basic weave patterns [9]. This paper further fine-tunes some of

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**Nomenclature**

$a$	flattened major diameter	$Q_{st}$	heat flow due to conduction from intersections of warp and weft
$A$	surface area in contact	$q_T$	heat flux due to radiation
$A_m$	total area of the unsupported length of warp or weft	$R$	thermal resistance, no. of threads in a weave repeat
$B$	flattened minor diameter	$T$	temperature
$c$	crimp in intersection region	$t$	thickness of fabric
$d$	circular thread diameter	$\beta$	back scatter fraction
$D$	sum of the diameters of warp and weft	$\delta$	constant defined by Eq. (8)
$e$	coefficient of yarn flattening	$\varepsilon$	emissivity
$F$	view factor	$\theta$	weave angle
$h'$	crimp amplitude in intersection region	$\sigma$	Stefan Boltzmann's constant
$k$	thermal conductivity	$\sigma_e$	extinction coefficient
$k_a$	thermal conductivity of air	$\sigma_s$	scattering coefficient
$L$	thickness of the fibrous web	$\tau_0$	optical thickness
$N$	number of radiating surfaces	1, 2	warp and weft, respectively; hot and cold surface, respectively
$p$	average thread spacing	$i, j$	radiating surfaces
$p'$	thread spacing in intersection region		
$Q$	heat flow		
$Q_a$	heat flow due to conduction from air pore		
$Q_m$	heat flow due to conduction from unsupported lengths of warp and weft		

the theories in radiation heat transfer through the air pores to make them more practically applicable. It was seen that the values predicted by the model for woven fabrics are very near to the actual values measured by a standard instrument in non-convective mode.

**2. Development of the model**

The woven fabric can be assumed to have a cellular geometry containing air pores, yarns between cross yarns and intersections of two yarns (Fig. 1). The yarns are porous in nature comprising of infinite length cylindrical fibres and air. If the fabric is kept in

a confined environment under a fixed pressure, the effects of forced convection can be neglected. The air inside the fabric can be assumed to be another insulating material. In this case, heat transfer takes place by conduction and radiation through the air pores and the porous yarns.

*2.1. Conductive heat transfer through fabric*

From the fabric geometry presented in Fig. 1(b and c), it can be seen that the conductive heat transfer through any basic weave takes place through (i) unsupported warp and weft yarns (ii) inter-laced region and (iii) air pore (only in case of non-convective

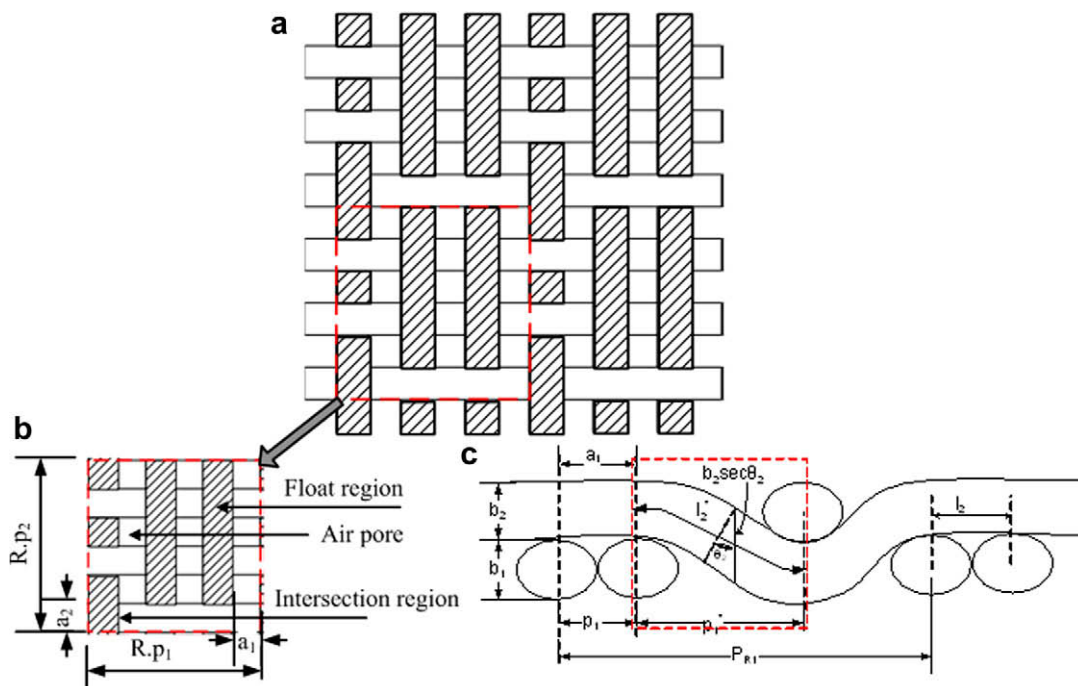


Fig. 1. Fabric structure with repeat unit. (a) Fabric with 1/2 weave; (b) close up of the repeat unit; (c) geometry of the weave.

mode). The total heat transfer due to conduction can be represented by

$$Q_{conduction} = Q_{m1} + Q_{m2} + Q_{st} + Q_a \quad (1)$$

The basic equation of conduction heat transfer is given by Fourier's equation integrated over a time  $t$ ,  $Q_{m1}$  can be calculated as

$$Q_{m1} = \frac{A_{m1} \cdot \Delta T}{\left(\frac{b_1 \sec \theta_1}{k_1} + \frac{t-b_1 \sec \theta_1}{k_a}\right)} \quad \text{where } A_{m1} = R^2 a_1 (p_2 - a_2) \quad (2)$$

Similarly  $Q_{m2}$ ,  $Q_{st}$ ,  $Q_a$  are given by

$$Q_{m2} = \frac{A_{m2} \cdot \Delta T}{\left(\frac{b_2 \sec \theta_2}{k_2} + \frac{t-b_2 \sec \theta_2}{k_a}\right)}, \quad (Q_{st} = \frac{A_{st} \cdot \Delta T}{\left(\frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{t-b_1-b_2}{k_a}\right)}, \quad \text{and} \quad (3)$$

$$Q_a = \frac{A_a \cdot \Delta T}{\left(\frac{t}{k_a}\right)}$$

where  $A_{m2} = R^2 a_2 (p_1 - a_1) A_{st} = R^2 a_1 a_2 A_a = R^2 (p_1 - a_1) (p_2 - a_2)$  Finally, the thermal resistance per unit area of the whole system to conduction can be given as

$$R_{conduction} = \frac{\Delta T \cdot (R^2 p_1 p_2)}{Q_{conduction}} \text{ K} \cdot \text{m}^2 / \text{W} \quad (4)$$

The flattened major and minor diameters, 'a' and 'b', respectively, can be calculated considering the Peirce's geometry in the intersection region [9] wherein the coefficient of flattening 'e' is given by

$$e = \frac{h_1'' + h_2''}{d_1 + d_2} = \frac{\frac{4}{3} [p_2'' \sqrt{c_1''} + p_1'' \sqrt{c_2''}]}{D} \quad (5)$$

2.2. Radiative heat transfer through air pore

The air pore (gap) can be assumed to be enclosed on four sides by yarns, bottom side by skin and top side, in absence of convection, a still black body (Fig. 2a and 2b). Calculations for enclosure radiation through air pore are performed using net radiation formulation proposed by Hottel [10]. The basic equation is given as

$$\sum_{j=1}^N \left( \frac{\delta_{kj}}{\epsilon_i} - F_{k-j} \frac{1 - \epsilon_j}{\epsilon_j} \right) \frac{Q_j}{A_j} = \sum_{j=1}^N (\delta_{kj} - F_{k-j}) \sigma T_j^4 \quad (6)$$

In the above equation subscripts represent surfaces,  $\delta_{kj}$  is defined such that

$$\delta_{kj} = \begin{cases} 1 & \text{when } k = j \\ 0 & \text{when } k \neq j \end{cases} \quad (7)$$

Fig. 2(b) shows an enclosure consisting of six surfaces. The lower surface ( $j = 1$ ) is the hot plate and the upper surface ( $j = 6$ ) is the atmosphere whose temperatures  $T_1$  and  $T_6$ , respectively, are known. Eq. (6) can be used to calculate the heat flows between the skin and the atmosphere. The equation can be converted into the following matrix form which can be solved with the help of matrix inversion method,

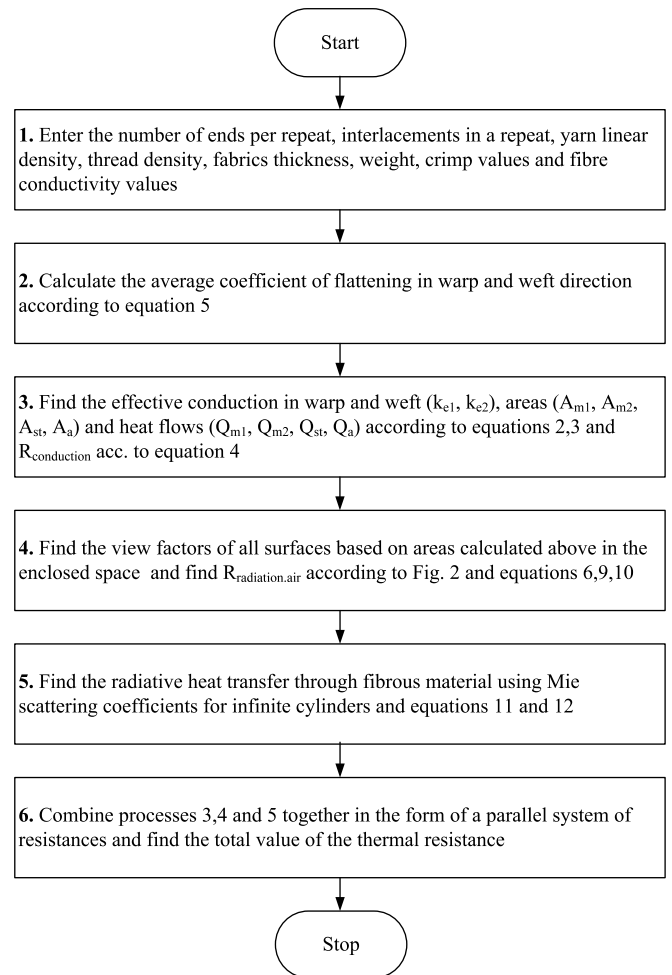


Fig. 3. Flow chart to calculate the thermal resistance of woven fabrics based on their constructional parameters.

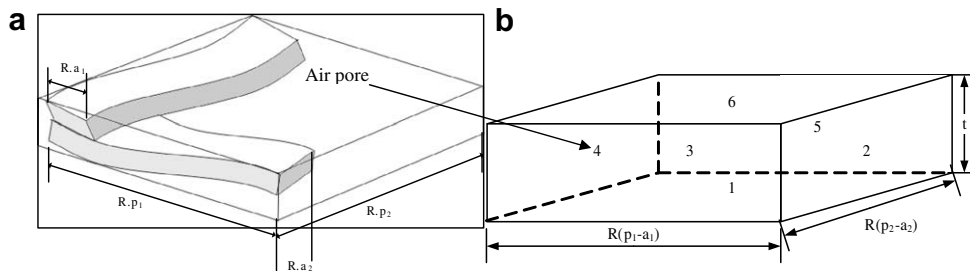


Fig. 2. Presence of air pore in a repeat. (a) Actual air pore; (b) the air pore as an enclosed surface used for net radiation method.

$$\begin{bmatrix} \frac{1}{\epsilon_1 A_1} & F_{12}\sigma & F_{13}\sigma & F_{14}\sigma & F_{15}\sigma & \frac{-F_{16}(1-\epsilon_6)}{A_6 \epsilon_6} \\ \frac{-F_{21}(1-\epsilon_1)}{\epsilon_1 A_1} & -\sigma & F_{23}\sigma & F_{24}\sigma & F_{25}\sigma & \frac{-F_{26}(1-\epsilon_6)}{A_6 \epsilon_6} \\ \frac{-F_{31}(1-\epsilon_1)}{\epsilon_1 A_1} & F_{32}\sigma & -\sigma & F_{34}\sigma & F_{35}\sigma & \frac{-F_{36}(1-\epsilon_6)}{A_6 \epsilon_6} \\ \frac{-F_{41}(1-\epsilon_1)}{\epsilon_1 A_1} & F_{42}\sigma & F_{43}\sigma & -\sigma & F_{45}\sigma & \frac{-F_{46}(1-\epsilon_6)}{A_6 \epsilon_6} \\ \frac{-F_{51}(1-\epsilon_1)}{\epsilon_1 A_1} & F_{52}\sigma & F_{53}\sigma & F_{54}\sigma & -\sigma & \frac{-F_{56}(1-\epsilon_6)}{A_6 \epsilon_6} \\ \frac{-F_{61}(1-\epsilon_1)}{\epsilon_1 A_1} & F_{62}\sigma & F_{63}\sigma & F_{64}\sigma & F_{65}\sigma & \frac{1}{\epsilon_6 A_6} \end{bmatrix} \begin{bmatrix} Q_1 \\ T_2^4 \\ T_3^4 \\ T_4^4 \\ T_5^4 \\ Q_6 \end{bmatrix} = \begin{bmatrix} \sigma T_1^4 & -F_{16}\sigma T_6^4 \\ -F_{21}\sigma T_1^4 & -F_{26}\sigma T_6^4 \\ -F_{31}\sigma T_1^4 & -F_{36}\sigma T_6^4 \\ -F_{41}\sigma T_1^4 & -F_{46}\sigma T_6^4 \\ -F_{51}\sigma T_1^4 & -F_{56}\sigma T_6^4 \\ -F_{61}\sigma T_1^4 & \sigma T_6^4 \end{bmatrix} \quad (8)$$

Once the unknown values are obtained, the total net radiation heat exchange between the hot plate and the atmosphere becomes equal to

$$Q_{radiation} = Q_1 - Q_6 \quad (9)$$

The resistance due to radiation through the air gap per unit area is given by

$$R_{radiation,air} = \frac{\Delta T \cdot R^2 p_1 p_2}{Q_{radiation}} \quad (10)$$

### 2.3. Radiative heat transfer through yarns

Yarn is a homogenous porous material consisting of fibres and air; hence radiation heat transfer is treated as radiation through porous media. The modular lengths of warp and weft and the intersecting portion in a repeat can be simplified into a fibrous web occupying the same area and volume with thickness as the weighted mean of the thicknesses of the warp, weft and the inter-

section point. The linear anisotropic scattering model for light-weight insulations in moderate temperature proposed by Tong and Tien [11,12] was used for the present case. Radiative heat flux per unit area can be calculated with the following equation

$$q_T = \frac{\sigma(T_1^4 - T_2^4)}{(1 + \gamma\tau_0)} \quad (11)$$

where  $\gamma = \frac{(3-\omega\alpha)}{4}$ ,  $\omega = \frac{\sigma_s}{\sigma_e}$ ,  $\alpha = 2(1 - 2\beta)$  and  $\tau_0 = \sigma_e L$ .

The calculations to find the scattering and extinction coefficients has been given by one of the authors [13]. Once the value of  $q_T$  is calculated, the resistance to thermal radiation through yarns is given by

$$R_{radiation,yarn} = \frac{\Delta T}{q_T} \quad (12)$$

### 2.4. Total thermal resistance

The total heat lost from the fabric can be considered to be analogous system of parallel resistances between the skin and the atmosphere. Hence the total fabric resistance can be given by the following equation:

$$R_{total} = [R_{conduction}^{-1} + R_{radiation,air}^{-1} + R_{radiation,yarn}^{-1}]^{-1} \quad (13)$$

Basic fabric parameters like yarn linear density, thread spacing, thickness, yarn crimp and fabric weight are used as inputs. Based on these values, the thermal resistance of the fabric can be calculated based on the flow chart given in Fig. 3.

## 3. Validation with experimental values

The model described in the previous section can be used to predict the thermal insulation of woven fabrics in a non-convective mode of heat transfer This situation is attainable in a standard

**Table 1**  
Properties of fabrics tested for thermal resistance.

Fabric Ref.	Weave	Warp count (Ne)	Weft count (Ne)	Ends (m)	Picks (m)	Thickness (mm)	Fabric weight (g/m <sup>2</sup> )	Porosity	Thermal resistance (K m <sup>2</sup> /W)
37	3/1 twill	41	36	5920	4160	0.44	151	0.680	0.0097
40	3/1 twill	38	27	5920	3840	0.32	169	0.641	0.0061
48	3/1 twill	6	6	2560	1600	0.95	534	0.600	0.0133
24	Plain	39	39	5680	2240	0.37	173	0.644	0.0075
25	Plain	38	38	5600	3200	0.28	151	0.619	0.0060
26	Plain	39	38	5600	3760	0.23	144	0.570	0.0049
29	Plain	37	19	5440	2720	0.31	161	0.658	0.0064
30	Plain	37	20	5600	2640	0.29	166	0.635	0.0059
52	2/1 twill	9	13	2880	1840	0.42	292	0.630	0.0077
75	3/1 twill	20	16	4960	2560	0.24	236	0.612	0.0036
83	4 end satin	30	10	6560	1920	0.35	255	0.627	0.0048
87	3/1 twill	20	20	4400	2320	0.32	210	0.659	0.0047
8	3/1 twill	2/40	2/38	4960	2320	0.49	217	0.683	0.0090
42	3/1 twill	2/34	2/32	5040	3040	0.45	245	0.657	0.0074
44	3/1 twill	2/36	2/26	5040	2720	0.50	265	0.630	0.0091
46	3/1 twill	2/38	2/54	5120	3680	0.38	228	0.635	0.0070
13	2/2 twill	2/40	2/38	4800	2240	0.46	215	0.671	0.0079
15	2/2 twill	2/40	2/40	4880	2080	0.49	211	0.684	0.0091
18	4 end satin	2/40	2/40	4880	2480	0.47	217	0.668	0.0085
20	4 end satin	2/40	2/40	4960	2000	0.42	209	0.701	0.0070
76	3/1 twill	2/50	2/50	6000	2800	0.27	220	0.614	0.0039
78	3/1 twill	2/40	2/20	4800	2400	0.37	300	0.571	0.0056
79	Plain	2/40	2/40	4800	2400	0.24	240	0.542	0.0031
84	Plain	2/40	2/40	4080	2960	0.21	215	0.558	0.0031
38	3/1 twill	39	2/36	5600	3440	0.39	191	0.661	0.0075
39	3/1 twill	37	2/36	6000	3360	0.38	196	0.651	0.0071
41	3/1 twill	42	2/70	5920	4160	0.29	155	0.623	0.0058
17	4 end satin	2/38	19	4640	2000	0.50	208	0.703	0.0091
27	Plain	36	2/70	5760	3680	0.22	145	0.603	0.0047

instrument that measures the thermal properties of fabrics called ALAMBETA where the fabric sample is placed between two plates and the measurement is carried out in a non-convective mode [8]. The values predicted by the model were compared with the actual values for the same set of fabric data obtained from ALAMBETA.

3.1. Materials

Fabrics of different weave structures, warp and weft counts, thread spacing, thickness and fabric weight were considered and their thermal resistance measured in ALAMBETA. The structural parameters of the fabrics are given in Table 1. All these fabrics were made of cotton so the fibre conductivity remains constant at 0.06 W/m K.

3.2. Comparison of theoretical and experimental values

Table 2 gives a comparison of the values obtained from the experiment and the model. The average error percentage between the actual and the predicted values is 12%. This can be attributed to the inherent variations in the textile structure in terms of hairiness and compactness and also the measurement technique of the instrument explained later. Fig. 4 gives the scatter of the data points. It can be seen that the coefficient of determination value ( $r^2$ ) is high (0.86) which indicates a strong correlation between the actual and the predicted values of thermal resistance. The equation represented by the trend line can also be utilized to obtain a close estimate of the actual thermal resistance. The comparison of individual predicted values of thermal resistances and those obtained experimentally from ALAMBETA is given in Fig. 5. In some cases, the values of thermal resistance obtained theoretically are

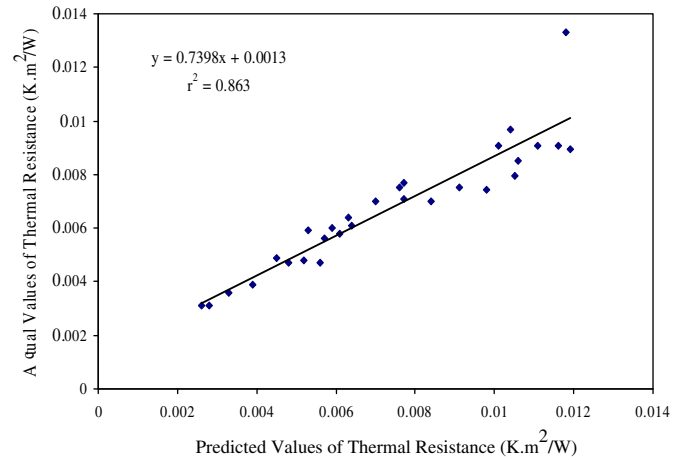


Fig. 4. Correlation between the actual values of thermal resistance obtained from ALAMBETA and the predicted values from the model.

marginally higher than the actual values of ALAMBETA. This is because the pressure applied by the top plate tends to flatten the yarns and this leads to an increase in the conductive heat losses. Therefore, the actual value of resistance is lesser than the predicted value.

4. Conclusions

An approach to calculate the total heat transfer through woven fabrics based on the theories of heat transfer has been discussed. It was seen that all the basic weaves can be represented as a repeat unit consisting of stack of yarns, unsupported yarns

Table 2 Comparison between experimental and theoretical values.

Fabric Ref.	Actual thermal resistance (A) (K m <sup>2</sup> /W)	Heat flow by conduction (Q <sub>cond</sub> )	Heat flow by radiation (air) (Q <sub>rad,air</sub> )	Heat flow by radiation (yarn) (Q <sub>rad,yarn</sub> )	Actual thermal resistance (P) (K m <sup>2</sup> /W)	Error $\frac{ A-P }{A}$
37	0.0097	740.82	160.08	62.75	0.0104	0.072
40	0.0061	1157.40	352.70	62.70	0.0064	0.049
48	0.0133	358.14	431.28	60.84	0.0118	0.112
24	0.0075	881.34	379.03	62.86	0.0076	0.013
25	0.006	1293.30	331.36	62.83	0.0059	0.016
26	0.0049	1684.80	458.08	62.86	0.0045	0.081
29	0.0064	931.03	242.65	62.64	0.0063	0.015
30	0.0059	1306.80	518.66	62.77	0.0053	0.101
52	0.0077	813.04	0.00	62.50	0.0077	0.000
75	0.0036	1499.80	0.00	62.69	0.0033	0.083
83	0.0048	1952.50	0.00	62.74	0.0052	0.083
87	0.0047	1022.00	0.00	62.80	0.0056	0.191
8	0.0089	779.29	0.00	62.64	0.0119	0.328
42	0.0074	962.22	0.00	62.18	0.0098	0.317
44	0.0090	797.39	0.00	62.07	0.0116	0.276
46	0.0069	1124.20	0.00	62.50	0.0084	0.201
13	0.0079	888.84	0.00	62.74	0.0105	0.324
15	0.0090	837.71	0.00	62.78	0.0111	0.225
18	0.0085	1366.80	0.00	62.72	0.0106	0.245
20	0.0069	1034.20	0.00	62.91	0.007	0.001
76	0.0039	1850.80	0.00	62.79	0.0039	0.000
78	0.0056	3804.30	0.00	62.52	0.0057	0.017
79	0.0031	2553.50	0.00	62.96	0.0026	0.169
84	0.0031	1366.80	0.00	62.87	0.0028	0.096
38	0.0075	933.03	101.40	62.53	0.0091	0.213
39	0.0071	981.48	246.74	62.50	0.0077	0.084
41	0.0058	1253.00	310.33	62.80	0.0061	0.051
17	0.0090	927.05	0.00	62.85	0.0101	0.114
27	0.0047	2013.10	0.00	62.87	0.0048	0.019
					Average	0.12
					max	0.32
					min	0.00

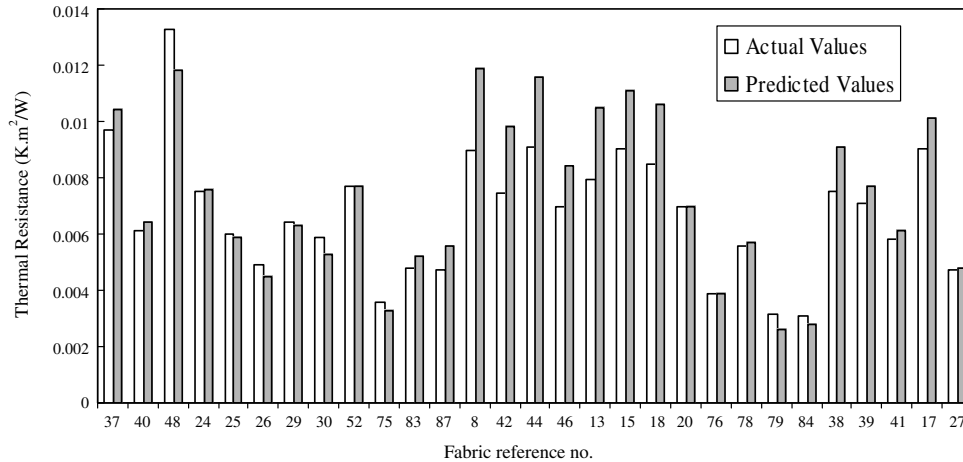


Fig. 5. Comparison of individual values of thermal resistance obtained from experimental data and the theoretical model.

between interlacements and air pores. Conductive heat transfer can be calculated using a parallel system method wherein the heat flow takes place through the air pore and the yarns. Radiation heat transfer can be segregated into two parts, one through the air pore and the other through the yarns. The air pore can be assumed to be an enclosure with six surfaces and net-radiation method can be used to find the radiative heat exchange between the hot surface and the atmosphere. Radiative heat transfer through yarns can be modeled in terms of linear anisotropic scattering where the yarns can be represented as a fibrous batting. The values of thermal resistance obtained from the mathematical model developed gives excellent prediction of the thermal resistance values when compared with experimental data.

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